CLASS:10TH DATE: 18-01-2021 SUB.:MATHEMATICS

1. .Check whether 6^n can end with the digit O for any natural number n.

Solution: If the number 6^n, for any n, were to end with the digit zero,

That is, the prime factorisation of 6^n would contain the prime 5.

But $6^n = (2 \times 3)^n = 2^n \times 3^n$ So the primes in factorisation of 6^n are 2 and 3.

So the uniqueness of the Fundamental Theorem of Arithmetic guarantees that (here are no other primes except 2 and 3 in the factorisation of 6^n.

So there is no natural number n for which 6” ends with digit zero.

Question 1.

Show that there is iw positive integer n for which $\sqrt{n+1} + \sqrt{n-1}$ is rational.

Solution:Let there be a positive integer n for which $\sqrt{n+1} + \sqrt{n-1}$ be rational number.

$$\sqrt{n-1} + \sqrt{n+1} = \frac{p}{q}\text{; where } p, q \text{ are integers and } q \neq 0 \quad \ldots(i)$$

$$\Rightarrow \frac{1}{\sqrt{n-1}+\sqrt{n+1}} = \frac{q}{p} \qquad \Rightarrow \frac{\sqrt{n-1}-\sqrt{n+1}}{\left(\sqrt{n-1}+\sqrt{n+1}\right)\times\left(\sqrt{n-1}-\sqrt{n+1}\right)} = \frac{q}{p}$$

$$\Rightarrow \frac{\sqrt{n-1}-\sqrt{n+1}}{(n-1)-(n+1)} = \frac{q}{p} \qquad \Rightarrow \frac{\sqrt{n-1}-\sqrt{n+1}}{n-1-n-1} = \frac{q}{p}$$

$$\Rightarrow \frac{\sqrt{n+1}-\sqrt{n-1}}{2} = \frac{q}{p} \qquad \Rightarrow \sqrt{n+1}-\sqrt{n-1} = \frac{2q}{p} \quad \ldots(ii)$$

Adding (i) and (ii), we get

$$\sqrt{n-1}+\sqrt{n+1}+\sqrt{n+1}-\sqrt{n-1} = \frac{p}{q}+\frac{2q}{p}$$

$$\Rightarrow 2\sqrt{n+1} = \frac{p^2+2q^2}{pq} \qquad \Rightarrow \sqrt{n+1} = \frac{p^2+2q^2}{2pq}$$

$\Rightarrow$ $\sqrt{n+1}$ is rational number as $\frac{p^2+2q^2}{2pq}$ is rational

$\Rightarrow$ $\sqrt{n+1}$ is perfect square of positive integer ...(A)

Again subtracting (ii) from (i), we get

$$\sqrt{n-1}+\sqrt{n+1}-\sqrt{n+1}+\sqrt{n-1} = \frac{p}{q}-\frac{2q}{p} \Rightarrow 2\sqrt{n-1} = \frac{p^2-2q^2}{pq}$$

$\Rightarrow$ $\sqrt{n-1}$ is rational number as $\frac{p^2-2q^2}{2pq}$ is rational.

$\Rightarrow\sqrt{n-1}$ is also perfect. square of positive integer From (A) and (B)

$\sqrt{n-1}$ and $\sqrt{n+1}$are perfect squares of positive integer. It contradicts the fact that two perfect squares differ at least by 3.

Hence, there is no positive integer n for which $\sqrt{n+1}$ + $\sqrt{n-1}$ is rational.

Question 2. Let a, b, e, k be rational numbers such that k is not a perfect cube. if a + $bk^{1/2}$ + $ck^{2/3}$ then prove that a = b, c = 0.

Solution:

Given, $$a + bk^{1/3} + ck^{2/3} = 0 \qquad ...(i)$$

Multiplying both sides by $k^{1/3}$, we have

$$ak^{1/3} + bk^{2/3} + ck = 0 \qquad ...(ii)$$

Multiplying (i) by b and (ii) by c and then subtracting, we have

$$\Rightarrow \quad (ab + b^2k^{1/3} + bck^{2/3}) - (ack^{1/3} + bck^{2/3} + c^2k) = 0$$

$$\Rightarrow \quad (b^2 - ac)k^{1/3} + ab - c^2k = 0$$

$$\Rightarrow \quad b^2 - ac = 0 \text{ and } ab - c^2k = 0 \qquad [\text{Since } k^{1/3} \text{ is irrational}]$$

$$\Rightarrow \quad b^2 = ac \text{ and } ab = c^2k$$

$$\Rightarrow \quad b^2 = ac \text{ and } a^2b^2 = c^4k^2$$

$$\Rightarrow \quad a^2(ac) = c^4k^2 \qquad [\text{By putting } b^2 = ac \text{ in } a^2b^2 = c^4k^2]$$

$$\Rightarrow \quad a^3c - k^2c^4 = 0 \quad \Rightarrow \quad (a^3 - k^2c^3)c = 0$$

$$\Rightarrow \quad a^3 - k^2c^3 = 0, \text{ or } c = 0$$

Now, $$a^3 - k^2c^3 = 0 \quad \Rightarrow \quad k^2 = \frac{a^3}{c^3}$$

$$\Rightarrow \quad \left(k^2\right)^{1/3} = \left(\frac{a^3}{c^3}\right)^{1/3} \quad \Rightarrow \quad k^{2/3} = \frac{a}{c}$$

This is impossible as $k^{2/3}$ is irrational and ac is rational.

$\therefore a^3 - k^2c^3 \neq 0$

Hence, c = 0

Substituting c = 0 in $b^2 - ac = 0$, we get b = 0

Substituting b = 0 and c = 0 in $a + bk^{1/3} + ck^{2/3} = 0$, we get a = 0

Hence, a = b = c = 0

Question 3.Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

Solution:It is given that on dividing 398 by the required number, there is a remainder of 7. This means that 398 – 7 = 391 is exactly divisible by the required timber In other words, required number is a factor of 391.

Similarly, required positive integer is a Íctor of 436 – 11 = 425 and 542 – 15 = 527

Clearly, the required number is the HCF of 391, 425 and 527.

Using the factor tree, we get the prime factorisations of 391, 425 and 527 as follows:

391 = 17 × 23, 425 52 × 17 and 527 17 × 31

∴ HCF of 391, 425, and 527 is 17.

Hence, the required number = 17.

2.